GammaGate

Your Gateway to GATE Excellence

GATE 2026 — Aerospace Engineering (AE)

Detailed Answer Key & Solutions — All 65 Questions

General Aptitude (Q1–Q10) · Core AE (Q11–Q65)

Exam GATE 2026 AE	**Sections** GA (Q1–Q10) + Core (Q11–Q65)	**Total Questions** 65
Question Types MCQ, MSQ (multi-select), NAT	**Marks** 1 or 2 marks	**Negative Marking** Nil for NAT/MSQ

GATE 2026 AE — Exam Analysis & Statistics

Overall Difficulty	**Total Questions**	**Total Marks**	**Duration**
Moderate to Hard	65	100 marks	3 hours
Exam Date	**Paper Code**	**Negative Marking**	**Calculator**
February 2026	AE	1/3 for MCQ; Nil for NAT/MSQ	Virtual

Section-wise Marks Distribution

Section	Topics Covered	Questions	Marks	Difficulty
General Aptitude	Verbal, Numerical, Logical	10	15	Easy–Moderate
Mathematics	Linear Algebra, ODEs, Calculus	7	11	Moderate
Flight Mechanics	Glide, Performance, Stability	6	10	Moderate
Aerodynamics	Potential Flow, Airfoils, Wings	10	17	Moderate–Hard
Structures	Elasticity, Torsion, Beams, Vibration	9	15	Hard
Propulsion	Turbomachinery, Rockets, Ramjets	12	20	Moderate–Hard
Space Dynamics	Orbits, Trajectories	4	7	Moderate
Gas Dynamics	Compressible Flow, Fanno, Rayleigh	7	5	Hard
Total		**65**	**100**	**Moderate–Hard**

Question Type Breakdown

Type	Description	Count	Marks Each
MCQ	Single correct; −1/3 negative marking	30	1 or 2
MSQ	Multiple correct; **no** negative marking	15	1 or 2
NAT	Numerical answer; **no** negative marking	20	1 or 2

GammaGate Expert Analysis

- **Structures & Gas Dynamics** were the toughest sections this year — Q58, Q59, Q65 required multi-step reasoning with no room for approximation.
- **Mathematics** was scoring — students who practiced ODEs and Linear Algebra well could have secured all 11 marks with confidence.
- **MSQ questions** (no negative marking) rewarded students who had conceptual clarity even without certainty — never leave these blank.
- **Aerodynamics** (Q25, Q29, Q47) tested deeper conceptual understanding rather than formula application — our Batch 2027 curriculum covers all such conceptual traps.
- **Expected cut-off:** General ≈ 33–37 marks | OBC ≈ 30–33 marks | SC/ST ≈ 22–25 marks (GammaGate estimate; official cut-off may vary)

General Aptitude (Q1–Q10)

Q1. "He often _____ the numbers. False claims are not going to help. Honesty _____ trust", said the manager. Choose the option with the correct order of words to fill the blanks.

(A) exaggerates; engenders

(B) excels; encourages

(C) aggravates; alleviates

(D) diminishes; eliminates

We need two verbs that make natural, meaningful sentences.

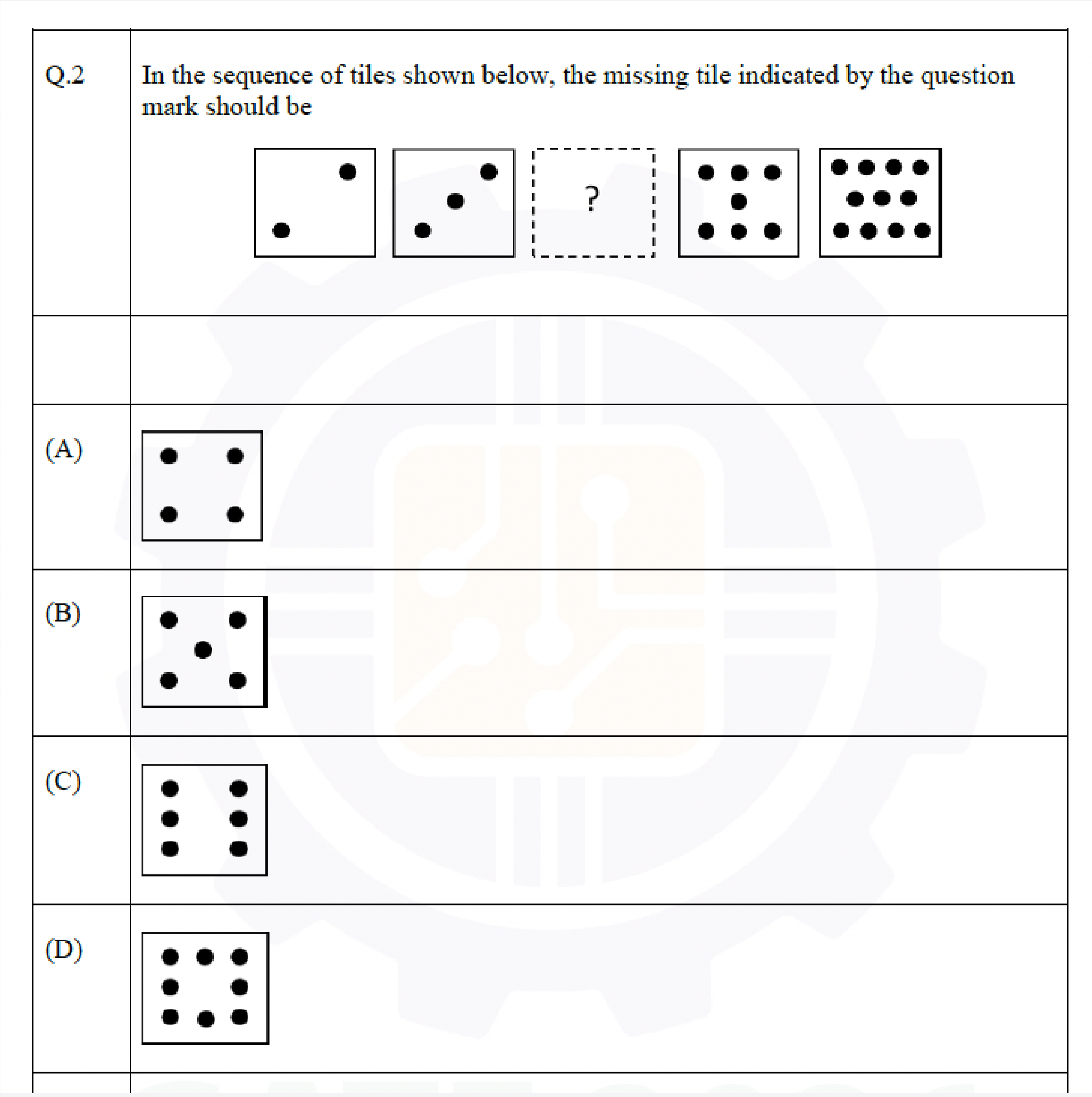
Answer: (A)

Why (A) is correct:

- "exaggerates the numbers" = overstates the figures (common usage).
- "Honesty engenders trust" = honesty *creates/builds* trust (standard collocation).

Why others are wrong:

- (B) "excels the numbers" is incorrect usage (you excel *in* something, not "excel the numbers").
- (C) "aggravates the numbers" is unnatural; and "honesty alleviates trust" is logically wrong (alleviate = reduce).
- (D) "diminishes the numbers" could mean reduces, but then "honesty eliminates trust" is the opposite of intended meaning.


Q.2 In the sequence of tiles shown below, the missing tile indicated by the question mark should be

?

(A)

(B)

(C)

(D)

Figure 1:

The given tiles have dot-counts: $2, 3, ?, 7, 11$. These are consecutive prime numbers:

$$2,\ 3,\ 5,\ 7,\ 11$$

So the missing tile must have **5 dots**.

Answer: (B)

Why others are wrong:

- (A) gives 4 (not prime in this sequence; breaks $2, 3, 5, 7, 11$).

- (C) gives 6 (not prime).
- (D) gives 8 (not prime).

Q3. A school has 100 students distributed among 1st to 10th standards. Which statement is always correct?

(A) There are at least 10 students who belong to the same standard.

(B) There is at least one student in each standard.

(C) There are at most 10 students in 10th standard.

(D) The total number of students from 1st to 5th standards is at least 50.

There are 10 standards (boxes) and 100 students (objects). By the pigeonhole principle, at least one standard has at least

$$\left\lceil \frac{100}{10} \right\rceil = 10$$

students.

Answer: (A)

Why others are wrong (counterexamples):

- (B) False: all 100 students could be in just one standard; then some standard has 0.
- (C) False: 10th standard could have 100 students.
- (D) False: students could be concentrated in standards 6–10, making total in 1–5 less than 50.

Q4. How many 3-digit numbers can be formed using three distinct single-digit prime numbers?

(A) 64

(B) 24

(C) 12

(D) 4

Single-digit primes are $\{2, 3, 5, 7\}$ (4 choices). We must pick 3 distinct primes and arrange them:

$$^4P_3 = 4 \times 3 \times 2 = 24$$

Answer: (B)

Why others are wrong:

- (A) 64 would resemble 4^3 (repetition allowed) but repetition is **not** allowed.
- (C) 12 would be $\binom{4}{3}$ (just choosing) but order matters for numbers.
- (D) 4 is far too small; that would be like choosing only the first digit.

Q5. In a group of students, 10 like Mathematics, 12 like English, 4 like both, and 6 like neither. The number of students in the group is ____.

(A) 18

(B) 20

(C) 24

(D) 32

Using inclusion–exclusion:

$$|M \cup E| = |M| + |E| - |M \cap E| = 10 + 12 - 4 = 18$$

Add those who like neither:

$$\text{Total} = 18 + 6 = 24$$

Answer: (C)

Why others are wrong:

- (A) 18 counts only those who like at least one subject, ignoring the 6 who like neither.
- (B) 20 would correspond to mistakenly subtracting/adding intersection incorrectly.
- (D) 32 would correspond to double-counting (e.g., adding 10 and 12 and also adding 6 without removing overlap).

Q6. Charity : P :: Retaliation : Q. Choose the appropriate pair of words P and Q.

(A) P = Parsimonious; Q = Vengeful

(B) P = Altruistic; Q = Amicable

(C) P = Resentful; Q = Spiteful

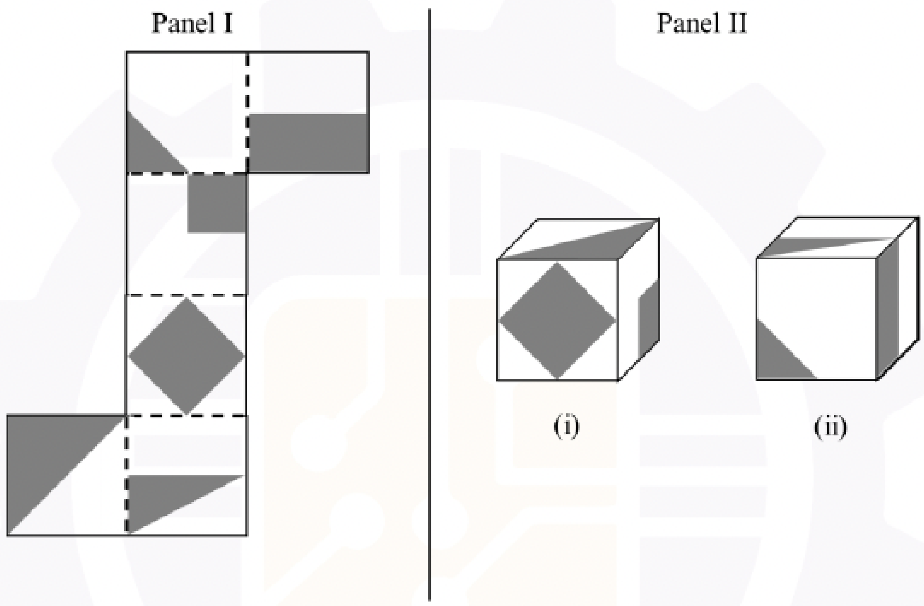
(D) P = Magnanimous; Q = Vindictive

Charity aligns with being **magnanimous** (large-hearted/generous). Retaliation aligns with being **vindictive** (seeking revenge).

Answer: (D)

Why others are wrong:

- (A) “Parsimonious” means stingy—opposite of charity (though “vengeful” fits retaliation).
- (B) “Altruistic” fits charity, but “amicable” means friendly—not retaliation.
- (C) Both words are negative; charity is not “resentful”.

Q.7	A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct? 
(A)	Only (i) can correspond to the unfolded cube in Panel I.
(B)	Only (ii) can correspond to the unfolded cube in Panel I.
(C)	Both (i) and (ii) can correspond to the unfolded cube in Panel I.
(D)	Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

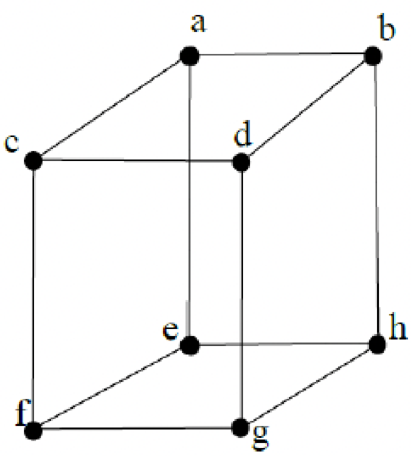
When folding a net, **adjacency and relative orientation** of faces is preserved. In the given net, the diamond face must end up adjacent to the specific shaded side faces in the only consistent orientation. Option (i) matches this adjacency/orientation; option (ii) violates it (a shaded region lands on an incompatible face relative to the diamond).

Answer: (A)

Why others are wrong:

- (B) contradicts the required adjacency of the diamond face to the shaded side face.
- (C) cannot be true because (ii) already violates adjacency/orientation.
- (D) cannot be true because (i) is consistent with the fold.

Q.8	Consider the cube shown below with its 8 corners labelled a, b, c, d, e, f, g, and h. The figure is representative. All corners are to be colored such that any two corners that are connected by an edge must be of different colors. The minimum number of colors required to achieve this is ________
(A)	8
(B)	4
(C)	3
(D)	2



The cube-corner graph is **bipartite**: corners can be split into two sets (like a 3D checkerboard) such that every edge connects opposite sets. Hence 2 colors suffice, and 1 color is impossible.

Answer: (D)

Why others are wrong:

- (A), (B), (C) are not minimal. Since 2 colors already work, needing 3/4/8 is false.

Q9. Four hills $H1, H2, H3, H4$ satisfy: (i) Neither $H2$ nor $H3$ is the easternmost. (ii) Neither $H2$ nor $H3$ is the westernmost. (iii) Neither the easternmost nor the westernmost is the southernmost. (iv) Two hills are to the west of $H2$. (v) The southernmost has at least two hills to its east. The southernmost hill is ____.

(A) $H1$

(B) $H2$

(C) $H3$

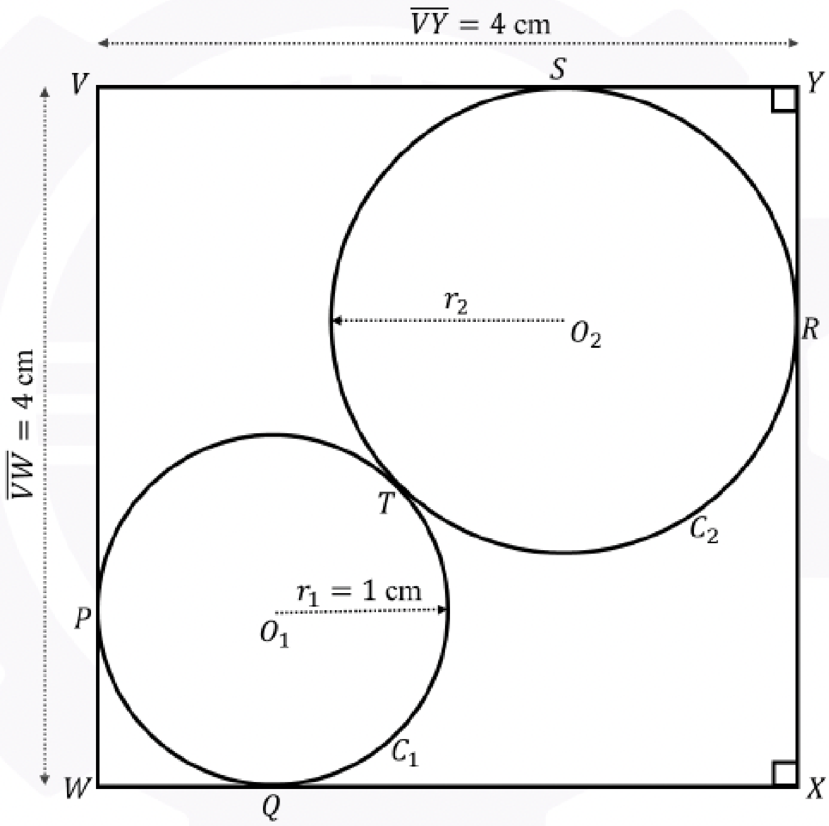
(D) $H4$

From (iv), exactly two hills lie west of $H2 \Rightarrow H2$ is **third from the west** (only one hill to its east).
From (i) and (ii), $H2$ and $H3$ are neither westernmost nor easternmost $\Rightarrow$ the extremes (westmost and eastmost) must be $\{H1, H4\}$.
From (v), the southernmost must have at least two hills to its east $\Rightarrow$ southernmost must be among the **two western positions**. But by (iii), the westernmost is **not** southernmost, so southernmost is the **second from the west**.
That position cannot be $H2$ (since $H2$ is third from west), hence it must be $H3$.

Answer: (C)

Why others are wrong:

- (B) $H2$ cannot be second from west (it must be third from west by (iv)).
- (A) and (D) are at the extremes (since extremes are $\{H1, H4\}$), but (iii) says extremes cannot be southernmost.

Q.10	As shown in the figure, circle C_1 with center O_1 and radius r_1 touches the square $VWXY$ at points P and Q while circle C_2 with center O_2 and radius r_2 touches the square $VWXY$ at points R and S. The two circles touch each other at T. Given $r_1 = 1$ cm and $\overline{VY} = \overline{VW} = 4$ cm, $r_2 =$ _____ cm. 
(A)	$4 - 3\sqrt{2}$
(B)	$1 + 2\sqrt{2}$
(C)	$7 - 4\sqrt{2}$
(D)	$5 + 3\sqrt{2}$

Place square corners at $(0,0)$ and $(4,4)$. Since C_1 touches left and bottom sides with $r_1 = 1$, its center is

$$O_1 = (1,1)$$

Since C_2 touches top and right sides, its center is

$$O_2 = (4 - r_2,\ 4 - r_2)$$

External tangency gives

$$O_1O_2 = r_1 + r_2 = 1 + r_2$$

Compute distance:

$$O_1O_2^2 = (3 - r_2)^2 + (3 - r_2)^2 = 2(3 - r_2)^2$$

So,

$$2(3 - r_2)^2 = (1 + r_2)^2$$

Take positive root:

$$\sqrt{2}(3 - r_2) = 1 + r_2$$
$$3\sqrt{2} - \sqrt{2}r_2 = 1 + r_2$$
$$r_2(1 + \sqrt{2}) = 3\sqrt{2} - 1$$
$$r_2 = \frac{3\sqrt{2} - 1}{1 + \sqrt{2}} = 7 - 4\sqrt{2}$$

Answer: (C)

Why others are wrong (quick numeric check):

$$7 - 4\sqrt{2} \approx 1.343$$

- (A) $4 - 3\sqrt{2} \approx -0.243$ (radius cannot be negative).
- (B) $1 + 2\sqrt{2} \approx 3.828$ (too large to fit touching top/right inside a 4×4 square).
- (D) $5 + 3\sqrt{2} \approx 9.243$ (impossible inside the square).

Aerospace Engineering (AE) : Q11–Q20 (1 mark each)

Compute the scalar curl (2D):

$$\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = \frac{\partial(2x + 4y)}{\partial x} - \frac{\partial(x + 2y)}{\partial y} = 2 - 2 = 0.$$

So $\vec{F}$ is conservative on a simply connected region, hence the closed line integral is zero:

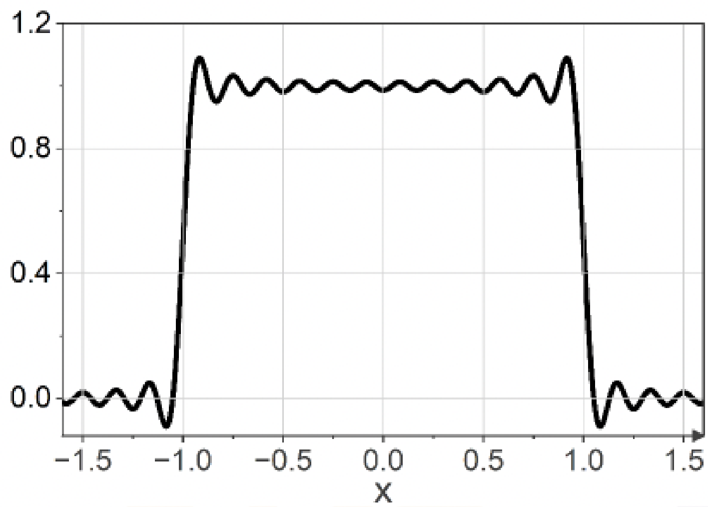
$$\oint_C \vec{F} \cdot d\vec{l} = 0.$$

(Indeed, a potential is $\phi = \frac{1}{2}x^2 + 2xy + 2y^2$, so $\nabla\phi = \vec{F}$.)

Answer: (A)

Why others are wrong: For any conservative field, *every* closed contour integral is exactly 0, so $(B), (C), (D)$ cannot occur.

Q.12	The Fourier series representation of a square wave is shown in the figure below. The fluctuations seen near x = ±1 are named after which one of the following scientists?



(A)	Cauchy
(B)	Fourier
(C)	Gibbs
(D)	Laplace

The overshoot/undershoot near jump discontinuities in Fourier series partial sums is called the **Gibbs phenomenon**.

Answer: (C)

Why others are wrong:

- (A) Cauchy is associated with complex analysis, convergence tests, etc., not this named overshoot.
- (B) Fourier introduced the series, but the *specific overshoot* is named after Gibbs.
- (D) Laplace is associated with transforms/probability; not this phenomenon.

Q13. The following equation with respect to $\phi(x, t)$, where a is a non-zero constant, represents ---------.

$$\frac{\partial \phi}{\partial t} + a\frac{\partial \phi}{\partial x} = 0$$

(A) linear wave propagation

(B) transient heat conduction

(C) Newton's law of cooling

(D) radiative transfer

This is the 1D **linear advection (transport)** equation. Its solution is

$$\phi(x, t) = f(x - at),$$

i.e., the profile propagates without distortion with speed a.

Answer: (A)

Why others are wrong:

- (B) heat conduction is typically $\phi_t = \alpha\phi_{xx}$ (second derivative in space).
- (C) Newton's cooling is an ODE like $T_t = -k(T - T_\infty)$.
- (D) radiative transfer has different governing forms (integro-differential), not this PDE.

Q14. Which one of the following makes an ideal air-standard Stirling cycle?

(A) Two reversible isobars, and two reversible adiabatics

(B) Two reversible isotherms, and two reversible isobars

(C) Two reversible isotherms, and two reversible isochores

(D) Isentropic compression, constant volume heat addition, isentropic expansion, and constant volume heat rejection

An ideal Stirling cycle consists of:

isothermal compression → constant-volume (regenerative) heat addition → isothermal expansion → constant-v

So it is **two isotherms + two isochores**.

Answer: (C)

Why others are wrong:

- (A) two isobars + two adiabatics corresponds to a Brayton-type structure.
- (B) two isotherms + two isobars is the Ericsson cycle (different from Stirling).
- (D) is the Otto cycle (two isochores + two isentropes, no isotherms).

Q15. In fluid dynamics, d'Alembert's paradox refers to which one of the following?

(A) Deviation of drag from $D \propto v^2$ at very low speeds

(B) Deviation of drag from $D \propto v^2$ at high subsonic speeds

(C) Prediction of zero drag by potential flow theory

(D) Presence of shocks in transonic flows

d'Alembert's paradox: inviscid, incompressible, irrotational (potential) flow around a body predicts **zero drag**, which contradicts real flows where viscosity causes drag.

Answer: (C)

Why others are wrong:

- (A),(B) are empirical deviations in drag scaling; not the paradox.
- (D) is compressible/transonic phenomenon unrelated to potential-flow zero-drag result.

Q16. The number of independent elastic constants that a fully anisotropic linear elastic material can have is ________.

(A) 36

(B) 21

(C) 10

(D) 2

General linear elasticity: $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$. With minor symmetries ($C_{ijkl} = C_{jikl} = C_{ijlk}$) and major symmetry ($C_{ijkl} = C_{klij}$), the stiffness reduces to a symmetric 6×6 matrix in Voigt notation. A symmetric 6×6 matrix has

$$\frac{6(6+1)}{2} = 21$$

independent entries.

Answer: (B)

Why others are wrong:

- (A) 36 would correspond to a general (not symmetric) 6×6 matrix, ignoring major symmetry.
- (C) 10 applies to certain restricted symmetries (not fully anisotropic).
- (D) 2 is for isotropic materials (e.g., E, ν or λ, μ).

Q17. A cantilever beam with an unsymmetric cross-section is subjected to a transverse shear force (P) at its free end. P acts at the shear center of the beam cross-section. Which one of the following statements is TRUE about the deformation of this beam?

(A) The beam undergoes only torsion

(B) The beam undergoes only bending

(C) The beam undergoes both torsion and bending

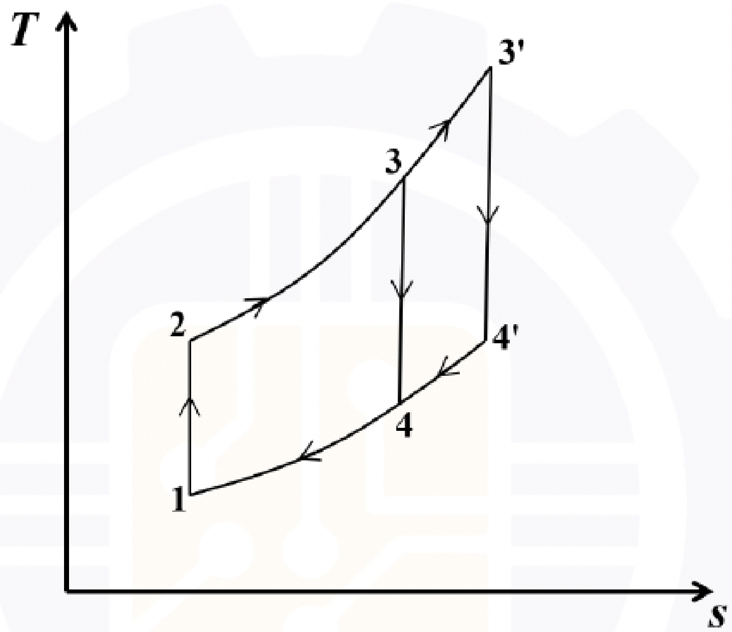
(D) The beam undergoes neither torsion nor bending

By definition, the **shear center** is the point through which a transverse load must pass to cause **no twisting** (no torsion) in an unsymmetric section. A transverse shear force still produces bending.

Answer: (B)

Why others are wrong:

- (A) At shear center, torsion is eliminated, not isolated.
- (C) Both torsion+bending occurs when the load does *not* pass through the shear center.
- (D) A transverse force at the free end certainly produces bending deformation.

Q.18	The figure below depicts two ideal gas turbine cycles, cycle 1-2-3-4-1 and cycle 1-2-3'-4'-1, on a ***T-s*** diagram. Which one the following statements is FALSE? 
(A)	The thermal efficiency of the two cycles is the same
(B)	The specific work of the two cycles is the same
(C)	The processes 2-3 and 2-3' are isobaric
(D)	The amount of heat added in the combustion process is greater for the cycle 1-2-3'-4'-1

Both cycles have the same compressor process 1-2 and (from the diagram) the same pressure ratio. For an **ideal Brayton cycle**, thermal efficiency depends only on pressure ratio (and γ), so (A) is **true**. Processes 2-3 and 2-3' are heat addition at (high) constant pressure, so (C) is **true**. Since point 3' is at a higher temperature than 3, the heat added for 2-3' exceeds that for 2-3, so (D) is **true**.
Specific net work equals the area enclosed by the cycle on the T-s diagram: the cycle reaching 3'

encloses a larger area, so the specific work is **not the same**. Hence statement (B) is **false**.

Answer: (B)

Why other options are not false:

- (A) True for ideal Brayton at fixed pressure ratio.
- (C) True: combustion/heat addition is modeled as isobaric.
- (D) True: higher turbine inlet temperature $\Rightarrow$ more heat addition.

Q19. The velocity potential function (ϕ) given below represents which one of the following?

$$\phi = 5x - 12y$$

(A) Doublet

(B) Irrotational vortex

(C) Source

(D) Uniform flow

Velocity components from potential:

$$u = \frac{\partial \phi}{\partial x} = 5, \qquad v = \frac{\partial \phi}{\partial y} = -12,$$

both constants $\Rightarrow$ a **uniform flow**.

Answer: (D)

Why others are wrong:

- (A) doublet potential varies like $\cos\theta/r$ (not linear in x, y).
- (B) vortex potential involves θ (multi-valued), not linear.
- (C) source potential involves $\ln r$, not linear.

Q20. The fundamental purpose of the Kutta condition in the thin airfoil theory is ________.

(A) to determine the total strength of the source distribution

(B) to determine the speed of the uniform flow

(C) to incorporate the essential effect of viscosity in the potential flow theory

(D) to incorporate the concept of induced drag in the inviscid theory

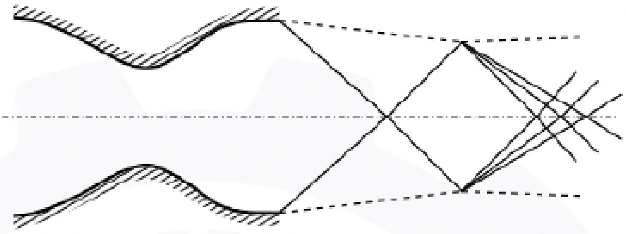
Thin airfoil theory is inviscid/potential flow, but the circulation is not uniquely determined unless we impose a physical condition. The **Kutta condition** enforces smooth flow leaving the sharp trailing edge (finite velocity at the trailing edge), which is a **viscosity-driven** selection principle (real flows shed vorticity to satisfy it). Thus it effectively incorporates the essential viscous effect needed to fix circulation.

Answer: (C)

Why others are wrong:

- (A) source distribution strength relates to thickness modeling; Kutta fixes circulation, not net source strength.
- (B) free-stream speed is an input, not determined by Kutta condition.
- (D) induced drag is a 3D finite-wing effect; Kutta is applied in 2D airfoil theory to fix circulation.

Aerospace Engineering (AE) : Q21–Q30 (1 mark each)

Q.21	In the figure shown below, the flow at the nozzle exit is ________. 
(A)	overexpanded
(B)	underexpanded
(C)	ideally expanded
(D)	subsonic

From the figure, oblique shock waves appear *right at/just after the nozzle exit.* That indicates the jet is being **compressed** to match a higher ambient pressure, i.e.

$$P_e < P_a \quad \Rightarrow \quad \text{overexpanded nozzle.}$$

Answer: (A)

Why others are wrong:

- (B) Underexpanded corresponds to $P_e > P_a$ and produces **expansion fans** at the exit (not shocks).
- (C) Ideally expanded means $P_e = P_a$ and there would be **no adjustment waves** (no shock/expansion pattern).
- (D) The presence of oblique shocks implies the external jet is **supersonic**; a subsonic exit cannot form oblique shocks.

Q22. An $n \times n$ square matrix A satisfies $A^T = A^{-1}$. The determinant of this matrix may take which of the following value(s)?

(A) $+1$

(B) -1

(C) n

(D) 0

Given $A^T = A^{-1}$, A is an **orthogonal** matrix. Taking determinants:

$$\det(A^T) = \det(A^{-1}).$$

But $\det(A^T) = \det(A)$ and $\det(A^{-1}) = \frac{1}{\det(A)}$. Hence

$$\det(A) = \frac{1}{\det(A)} \Rightarrow \det(A)^2 = 1 \Rightarrow \det(A) = \pm 1.$$

Answer(s): (A),(B)

Why others are wrong:

- (C) $\det(A) = n$ violates $\det(A) = \pm 1$ for orthogonal matrices.
- (D) $\det(A) = 0$ would make A singular, so A^{-1} would not exist (contradiction).

Q23. Which of the following statements is/are TRUE about the stability of an aircraft?

(A) Static stability of an aircraft is sufficient to guarantee its dynamic stability

(B) Static stability of an aircraft is related only to its initial tendency to return towards the equilibrium position from which it is disturbed

(C) An aircraft may be dynamically unstable even if it is statically stable

(D) Dynamic stability is related to the time history of aircraft motion after being disturbed from its equilibrium position

Static stability concerns the *initial tendency* (restoring vs. diverging) immediately after a disturbance. **Dynamic stability** concerns the *time evolution* (does the motion decay/grow with time).
Thus:

- (B) is true (definition of static stability).
- (D) is true (definition of dynamic stability).
- (C) is true: static stability does *not* guarantee dynamic stability (you can have oscillations that grow with time).

- (A) is therefore false.

Answer(s): (B),(C),(D)

Why (A) is wrong: Static stability is only a necessary tendency condition; dynamic behavior also depends on damping and system dynamics.

Q24. For a given air-standard power, the propulsive efficiency of a turbofan engine is more than that of a turbojet engine. Which of the following is/are the reason(s) for this?

(A) The mass flow rate is more for a turbofan engine

(B) The exit velocity is lower for a turbofan engine

(C) A turbofan engine operates at a lower altitude

(D) The fan of a turbofan engine consumes lesser power

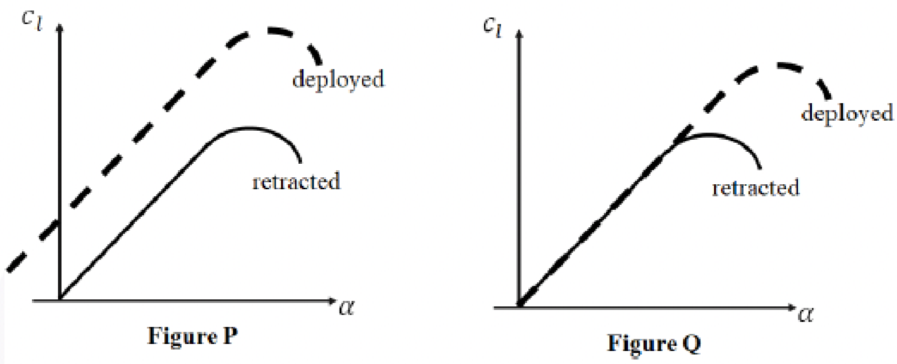
Propulsive efficiency improves when the jet velocity increment over freestream is smaller (less kinetic energy wasted in the exhaust). A turbofan produces the same thrust/power by accelerating a **larger mass flow** by a **smaller** ΔV.
Hence:

- (A) true: turbofan has larger mass flow (bypass stream).
- (B) true: turbofan typically has lower effective exhaust velocity for a given thrust/power.

Answer(s): (A),(B)

Why others are wrong:

- (C) Altitude is an operating choice/mission issue, not the fundamental reason for higher propulsive efficiency.
- (D) The fan *requires* shaft power; "consumes lesser power" is not a general truth and not the defining reason for higher propulsive efficiency.

Q.25	Shown below are qualitative illustrations of the lift curve for an airfoil when two different control surfaces are in their respective retracted and deployed configurations. Which of the following is/are TRUE? 
(A)	Figure P is for a flap
(B)	Figure P is for a slat
(C)	Figure Q is for a slat
(D)	Figure Q is for a flap

Flap deployment increases effective camber, shifting the lift curve *upward* (higher C_L at the same α), often with reduced stall angle. **Slat deployment** energizes the upper-surface flow and delays separation, increasing *stall angle* and $C_{L,\max}$ more than shifting the whole curve up. From the plots:

- Figure P shows a strong *upward shift* in C_L at a given $\alpha \Rightarrow$ flap.
- Figure Q shows stall delayed/higher α behavior $\Rightarrow$ slat.

Answer(s): (A),(C)

Why (B) and (D) are wrong: They swap the physical effects (flap vs. slat) seen in the curves.

Q26. The state of stress at a point in a 2-D body, in the x-y Cartesian coordinate system, is represented in matrix form as $[\sigma]$. The transformation matrix $[Q]$ rotates the coordinate system to a new x'-y' Cartesian coordinate system. Select the CORRECT option(s) that represent(s) the state of stress in the new coordinate system.

(A) $[Q]\,[\sigma]\,[Q]^T$

(B) $[Q]\,[\sigma]\,[Q]^{-1}$

(C) $([Q]^{-1})^T\,[\sigma]\,[Q]^T$

(D) $[Q]^{-1}\,[\sigma]\,[Q]$

For a second-order tensor under a **coordinate rotation** by an orthogonal matrix Q, the stress components transform as

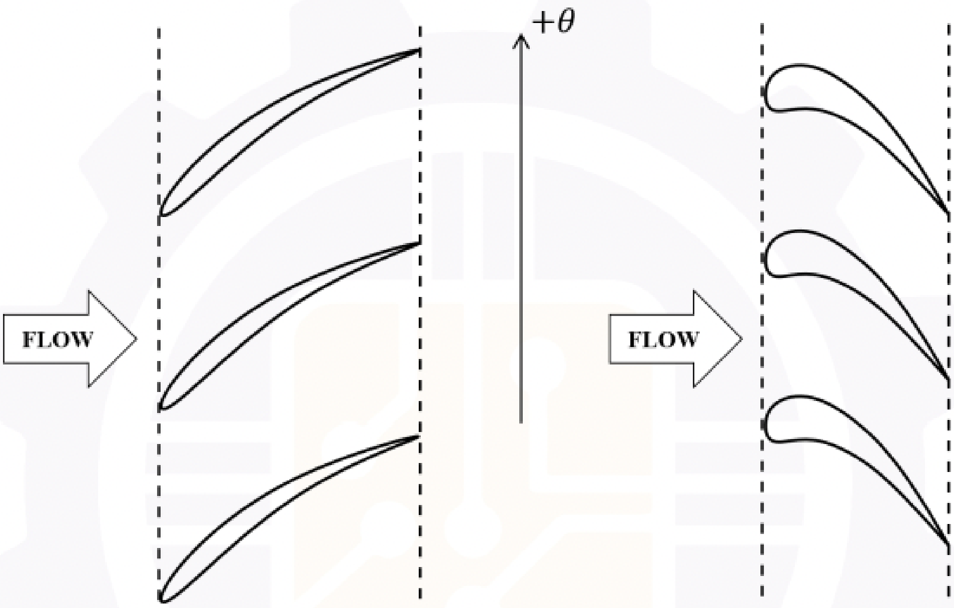
$$[\sigma]' = [Q]\,[\sigma]\,[Q]^T.$$

Since Q is a rotation matrix, it is orthogonal: $Q^{-1} = Q^T$ and $(Q^{-1})^T = Q$.
Therefore:

$$[Q]\,[\sigma]\,[Q]^T \equiv [Q]\,[\sigma]\,[Q]^{-1} \equiv ([Q]^{-1})^T\,[\sigma]\,[Q]^T.$$

Answer(s): (A),(B),(C)

Why (D) is wrong: $[Q]^{-1}[\sigma][Q]$ corresponds to the opposite convention/order; it is not the correct stress transformation for the stated rotation mapping.

Q.27	The figure below shows the blading of the rotors of two different axial turbomachines under their typical operating conditions, labelled as Configuration I and Configuration II. Which of the following statements is/are TRUE?  Configuration I Configuration II
(A)	Configuration I corresponds to the rotor of a compressor and Configuration II corresponds to the rotor of a turbine
(B)	Configuration I corresponds to the rotor of a turbine and Configuration II corresponds to the rotor of a compressor
(C)	The rotor blades of the turbomachine in Configuration I move along the $+\theta$ direction
(D)	The rotor blades of the turbomachine in Configuration II move along the $+\theta$ direction

Using typical rotor blade camber/orientation relative to flow:

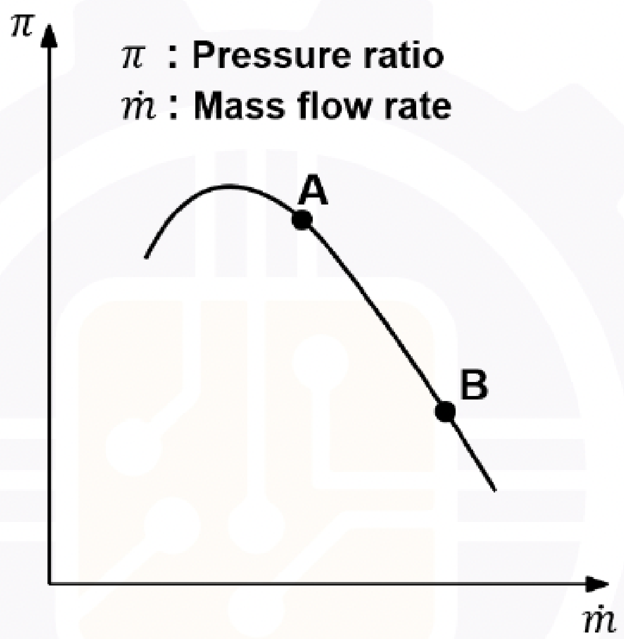
- Configuration I matches a **compressor rotor** (adds energy to flow).
- Configuration II matches a **turbine rotor** (extracts energy from flow).

Also, for the shown blade orientations, the turbine rotor in Configuration II corresponds to motion along $+\theta$.

Answer(s): (A),(D)

Why others are wrong:

- (B) swaps compressor/turbine identification.
- (C) gives the wrong sense of rotation for Configuration I based on the blade orientation shown.

Q.28	A multi-stage axial compressor can be operated at two points, A and B, both of which lie on the same speed line, as shown in the figure below. If η is the isentropic efficiency of the compressor, select the statements that is/are TRUE. 
(A)	$\eta_A > \eta_B$
(B)	$\eta_B > \eta_A$
(C)	In comparison to point B, point A is closer to the surge point
(D)	In comparison to point A, point B is closer to the choke point

On a compressor map along a given speed line:

- Moving left (lower mass flow) approaches **surge**; moving right (higher mass flow) approaches **choke**.
- The peak efficiency typically occurs near the **middle/near the "hump"** of the speed line; points deeper into the falling region generally have lower efficiency.

From the figure, A lies closer to the left side of the speed line than B, hence A is closer to surge and B is closer to choke. Also, A lies nearer the high-efficiency region than B, hence $\eta_A > \eta_B$.

Answer(s): (A),(C),(D)

Why (B) is wrong: It contradicts the usual efficiency trend shown by the speed line (A is nearer the efficient region than B).

Q29. Which of the following statements is/are TRUE regarding critical and drag divergence Mach numbers of a wing?

(A) Critical Mach number is the minimum freestream Mach number for which sonic condition is attained somewhere over the wing

(B) Drag divergence Mach number is always higher than the critical Mach number

(C) Drag divergence Mach number is the local Mach number over the wing at which the drag increases drastically

(D) Critical Mach number is independent of the angle of attack

- (A) is true by definition of M_{cr}.
- (B) is true: drag divergence M_{dd} occurs after/above M_{cr} (once transonic effects/shocks grow enough to cause rapid drag rise).

Answer(s): (A),(B)

Why (C) is wrong: M_{dd} is defined as a **freestream Mach number**, not the local Mach number on the wing. **Why (D) is wrong:** M_{cr} depends on loading and hence varies with angle of attack (higher α increases local acceleration, typically reducing M_{cr}).

Q30. A flow is steady, inviscid and one-dimensional, with no shaft work or body forces. Which of the following is/are possible under the given conditions?

(A) Oblique shocks

(B) Sound propagation

(C) Rayleigh flow

(D) Fanno flow

Given: steady, inviscid, 1D, no shaft work, no body forces.

- **Sound propagation** can be modeled as small-amplitude 1D inviscid waves (linear acoustics) $\Rightarrow$ possible.

- **Rayleigh flow** is 1D inviscid flow with **heat addition/removal** (no shaft work) $\Rightarrow$ possible under these assumptions.

Answer(s): (B),(C)

Why others are wrong:

- (A) Oblique shocks are inherently **2D** (flow deflection + shock angle), not 1D.
- (D) Fanno flow requires **wall friction** (viscous effects), which contradicts inviscid.

Aerospace Engineering (AE) : Q31–Q40

Q31. An aircraft starts gliding in power-off condition at an altitude of 4 km. Given that the maximum lift to drag ratio of the aircraft is 15, the maximum glide range that the aircraft can cover, measured along the ground, is ________ km (rounded off to the nearest integer).

For a steady glide, the maximum horizontal distance (glide range) is

$$R_{\max} = h\left(\frac{L}{D}\right)_{\max}.$$

Given $h = 4$ km and $(L/D)_{\max} = 15$,

$$R_{\max} = 4 \times 15 = 60 \text{ km}.$$

Answer: 60

Common mistakes to avoid: (i) Using D/L instead of L/D. (ii) Mixing meters and km (here both are already in km).

Q32. If a matrix can be written as $A = uv^T$, where both u and v are n-dimensional real-valued non-zero column vectors, then the rank of the matrix A is ________ (answer in integer).

$A = uv^T$ is an **outer product** of two non-zero vectors. Every column of A is a scalar multiple of u, hence the column space is 1-dimensional. Therefore,

$$\text{rank}(A) = 1.$$

Answer: 1

Audit note: Rank would drop to 0 only if u or v were the zero vector (but both are non-zero here).

Q33. The response $x(t)$ of a freely vibrating single degree of freedom underdamped system is given below. In the equation, A and ϕ are constants. The damping ratio of the system is __________ (rounded off to 3 decimal places).

$$x(t) = Ae^{-5t} \sin(10t + \phi)$$

For an underdamped SDOF system,

$$x(t) = Ae^{-\zeta\omega_n t} \sin(\omega_d t + \phi), \qquad \omega_d = \omega_n \sqrt{1-\zeta^2}.$$

Comparing with $Ae^{-5t} \sin(10t + \phi)$:

$$\zeta\omega_n = 5, \qquad \omega_n \sqrt{1-\zeta^2} = 10.$$

From $\omega_n = 5/\zeta$, substitute into the second:

$$\frac{5}{\zeta}\sqrt{1-\zeta^2} = 10 \Rightarrow \sqrt{1-\zeta^2} = 2\zeta \Rightarrow 1-\zeta^2 = 4\zeta^2 \Rightarrow \zeta^2 = \frac{1}{5}.$$

Hence $\zeta = 1/\sqrt{5} = 0.447213\ldots$

$$\boxed{\zeta \approx 0.447}$$

Answer: 0.447

Q34. The vortex shedding frequency behind a landing gear model is found to be 50 Hz when tested in a wind tunnel operating at 5 m/s. If the actual landing gear size is 10 times that of the model, and it is designed to operate at 50 m/s, then the expected vortex shedding frequency behind it is ________ Hz (rounded off to the nearest integer).

Use Strouhal number similarity:

$$St = \frac{fL}{V} = \text{constant}.$$

Thus,

$$\frac{f_m L_m}{V_m} = \frac{f_p L_p}{V_p}.$$

Given $f_m = 50$ Hz, $V_m = 5$ m/s, $L_p = 10L_m$, $V_p = 50$ m/s:

$$f_p = f_m \frac{V_p}{V_m}\frac{L_m}{L_p} = 50 \cdot \frac{50}{5} \cdot \frac{1}{10} = 50 \text{ Hz}.$$

Answer: 50

Common mistake: Scaling frequency directly with velocity alone and forgetting length scaling.

Q35. An elliptic wing has a span of 6 m and a planform area of 6 m^2. When generating a lift coefficient of 0.6, the induced drag it incurs is _______ $\times 10^{-3}$ (rounded off to 1 decimal place).

For an elliptic wing, Oswald efficiency factor $e = 1$. Aspect ratio:

$$AR = \frac{b^2}{S} = \frac{6^2}{6} = 6.$$

Induced drag coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{(0.6)^2}{\pi(1)(6)} = \frac{0.36}{6\pi} = 0.0190986\ldots$$

So,

$$C_{D_i} = 19.1 \times 10^{-3} \quad \text{(to 1 decimal place).}$$

Answer: 19.1

Q36. An earth satellite has the instantaneous position vector $\vec{r}$ and velocity vector $\vec{v}$ as given below. Here $\hat{p}$ and $\hat{q}$ denote the unit vectors along the x and y axes of the perifocal frame, respectively. Assume that the value of gravitational parameter is 398600 km^3/s^2. Which one of the following trajectories does the satellite follow?

$$\vec{r} = (8000\hat{p} + 9000\hat{q})\ \text{km} \qquad \vec{v} = (-6\hat{p} + 6\hat{q})\ \text{km/s}$$

(A) Circle

(B) Hyperbola

(C) Parabola

(D) Straight line

Compute radius magnitude:

$$r = \sqrt{8000^2 + 9000^2} = 12041.595\ \text{km}.$$

Speed magnitude:

$$v = \sqrt{(-6)^2 + 6^2} = \sqrt{72} = 8.4853\ \text{km/s}.$$

Escape speed at r:

$$v_{esc} = \sqrt{\frac{2\mu}{r}} = \sqrt{\frac{2(398600)}{12041.595}} = 8.1366\ \text{km/s}.$$

Since $v > v_{esc}$, the specific orbital energy is positive $\Rightarrow$ **hyperbolic** trajectory.

Answer: (B)

Why others are wrong:

- (A) Circle requires $v = v_{circ} = \sqrt{\mu/r}$ and the velocity perpendicular to radius; here v is much larger than v_{circ}.
- (C) Parabola corresponds to $v = v_{esc}$; here $v > v_{esc}$.
- (D) Straight line is not a Keplerian conic under central gravity (unless $\mu = 0$).

Q37. For the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, if the relation $a + b = c + d$ holds and $a, b, c, d \neq 0$, then which one of the following statements about A is **FALSE**?

(A) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector

(B) $\lambda = a + b$ is an eigenvalue

(C) $\lambda = d - b$ is an eigenvalue

(D) $\lambda = d + b$ is an eigenvalue

Given $a + b = c + d$, we have

$$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a + b \\ c + d \end{bmatrix} = (a + b) \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

So (A) is true and (B) is true with eigenvalue $\lambda_1 = a + b$.
Sum of eigenvalues equals trace:

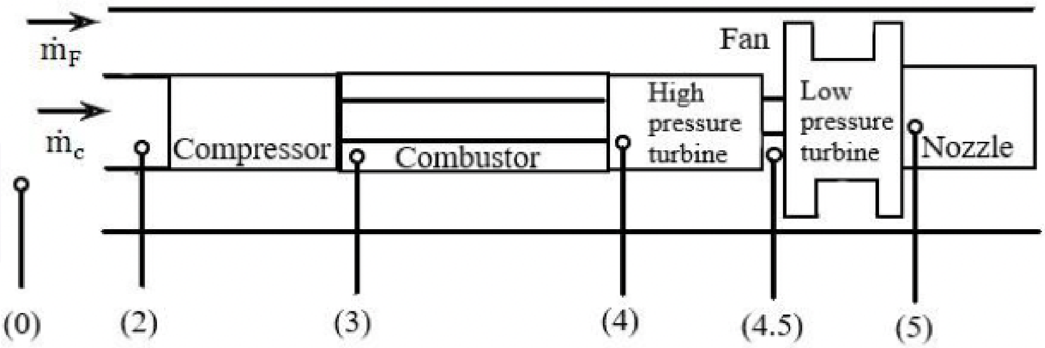
$$\lambda_1 + \lambda_2 = \mathrm{tr}(A) = a + d \Rightarrow \lambda_2 = (a + d) - (a + b) = d - b,$$

so (C) is also true. Therefore, the false statement is (D).

Answer: (D)

Why (D) is wrong: The second eigenvalue is forced to be $d - b$ (by trace), not $d + b$.

Q.38 In an ideal turbofan engine shown in the figure below, the compressor is driven by the high pressure turbine, and the fan is driven by the low pressure turbine. The stations 0, 2, 3, 4, 4.5, and 5 refer to free-stream, compressor inlet, compressor outlet, combustor exit, high pressure turbine exit, and low pressure turbine exit, respectively, and the subscript 't' refers to the total condition. Also, $\tau_r = T_{t0}/T_0$, $\tau_c = T_{t3}/T_{t2}$ and $\tau_\lambda = T_{t4}/T_0$. The total temperature ratio of the high pressure turbine $(T_{t4.5}/T_{t4})$ is given by ______.



(A)	$1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)$
(B)	$1 + \frac{\tau_r}{\tau_\lambda}(\tau_c + 1)$
(C)	$1 - \frac{\tau_r}{\tau_\lambda}(\tau_c + 1)$
(D)	$1 + \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)$

For an ideal engine, the high pressure turbine (HPT) provides exactly the compressor work (same core mass flow, same c_p):

$$c_p\,(T_{t4} - T_{t4.5}) = c_p\,(T_{t3} - T_{t2}) \;\Rightarrow\; \frac{T_{t4.5}}{T_{t4}} = 1 - \frac{T_{t3} - T_{t2}}{T_{t4}}.$$

Use the given ratios. For ideal inlet/diffuser, $T_{t2} = T_{t0} = \tau_r T_0$. Also $T_{t3} = \tau_c T_{t2} = \tau_c \tau_r T_0$ and

$T_{t4} = \tau_\lambda T_0$. Hence

$$\frac{T_{t3} - T_{t2}}{T_{t4}} = \frac{\tau_c \tau_r T_0 - \tau_r T_0}{\tau_\lambda T_0} = \frac{\tau_r}{\tau_\lambda}(\tau_c - 1).$$

Therefore

$$\frac{T_{t4.5}}{T_{t4}} = 1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1).$$

Answer: (A)

Why others are wrong: They have incorrect sign and/or the wrong $(\tau_c \pm 1)$ structure, which would imply HPT *adds* compressor work instead of supplying it.

Q39. An ideal rocket has characteristic exhaust velocity of 1200 m/s, mass flow rate of 75 kg/s, thrust coefficient of 1.5, and nozzle throat area of 0.025 m^2. The chamber pressure in kPa and the specific impulse due to gravity in seconds are ______, respectively. Assume that the acceleration due to gravity is 9.8 m/s^2.

(A) 3600 and 183.67

(B) 4600 and 190.51

(C) 3600 and 175.23

(D) 3500 and 183.67

Characteristic velocity:

$$c^* = \frac{p_c A_t}{\dot{m}} \Rightarrow p_c = \frac{\dot{m}c^*}{A_t} = \frac{75 \times 1200}{0.025} = 3.6 \times 10^6 \text{ Pa} = 3600 \text{ kPa}.$$

Thrust:

$$F = C_F p_c A_t.$$

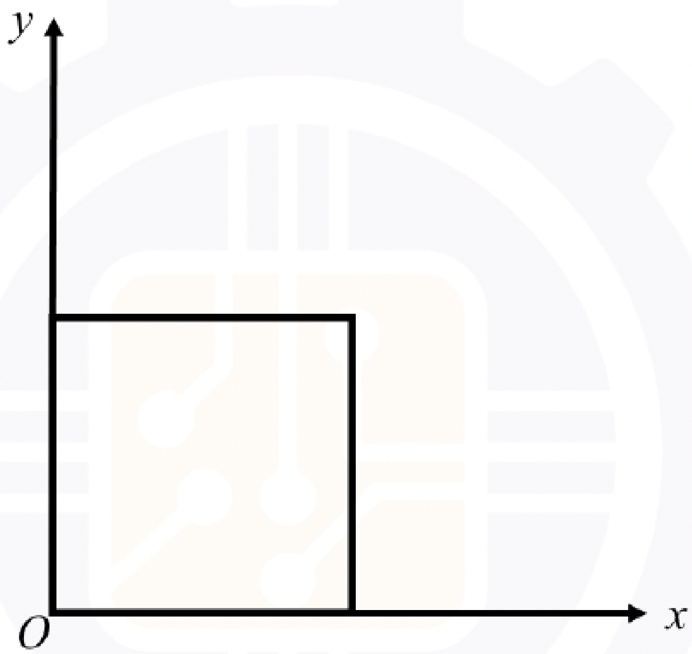
Specific impulse:

$$I_{sp} = \frac{F}{\dot{m}g_0} = \frac{C_F p_c A_t}{\dot{m}g_0} = \frac{C_F(\dot{m}c^*/A_t)A_t}{\dot{m}g_0} = \frac{C_F c^*}{g_0} = \frac{1.5 \times 1200}{9.8} = 183.67 \text{ s}.$$

Answer: (A)

Why others are wrong:

- (B) wrong p_c (does not satisfy $p_c = \dot{m}c^*/A_t$).
- (C) wrong I_{sp} (for ideal rocket, $I_{sp} = C_F c^*/g_0$ here).
- (D) wrong p_c (should be exactly 3600 kPa from the given data).

Q.40	Consider a unit square body as shown in the figure below. The body is subjected to the deformation field $u = -ay$ and $v = ax$, where 'a' is a constant. Due to the application of this deformation field, the body undergoes _______ in the x-y plane. 
(A)	biaxial deformation
(B)	pure shear
(C)	pure bending
(D)	rigid body rotation

Compute small strains:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 0, \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} = 0,$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = (-a) + a = 0.$$

All strain components are zero ⇒ no deformation (no change in lengths/angles). But the rotation is

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(a - (-a)\right) = a \neq 0,$$

so the motion is a **rigid body rotation**.

Answer: (D)

Why others are wrong:

- (A) would require non-zero ε_{xx} and/or ε_{yy}.
- (B) would require non-zero γ_{xy}.
- (C) would produce spatially varying normal strains (not all zero).

Aerospace Engineering (AE) : Q41–Q50

Q41. Consider a launch vehicle of mass 10 tons being launched vertically. The vehicle has 8 tons of propellant, which burns completely at a constant rate over 50 s. If the engine specific impulse is 250 s, and the acceleration due to gravity at sea level is g_0, the acceleration experienced by the vehicle at lift-off is __________.

(A) g_0

(B) $2g_0$

(C) $3g_0$

(D) $4g_0$

Propellant mass flow rate:

$$\dot{m} = \frac{8\text{ tons}}{50\text{ s}} = \frac{8000}{50} = 160\text{ kg/s}.$$

Thrust:

$$F = \dot{m} g_0 I_{sp} = 160\, g_0\, (250) = 40000\, g_0.$$

At lift-off, vehicle mass $m = 10$ tons $= 10000$ kg. Net acceleration:

$$F - mg_0 = ma \quad \Rightarrow \quad a = \frac{F - mg_0}{m} = \frac{40000g_0 - 10000g_0}{10000} = 3g_0.$$

Answer: (C)

Why others are wrong: (A) and (B) underestimate thrust-to-weight margin; (D) would require $F = 5mg_0$ at lift-off, which is not the case here.

Across the rotor (1→2): Rotor does work on the fluid ⇒ $T_{02} > T_{01}$ and ideally $p_{02} > p_{01}$.
Across the stator (2→3): Stator has *no shaft work* ⇒ $T_{03} \approx T_{02}$ (stagnation temperature ideally constant; losses mainly reduce total pressure). Due to losses, total pressure *drops* ⇒ $p_{03} < p_{02}$.
Also, stator acts as a diffuser (reduces absolute speed) ⇒ typically $C_3 < C_2$; and rotor generally increases absolute speed compared to upstream ⇒ $C_2 > C_1$ (for a typical compressor stage).
Thus the consistent set is:

$$p_{01} < p_{02},\ p_{02} > p_{03}; \quad T_{01} < T_{02},\ T_{02} \approx T_{03}; \quad C_1 < C_2,\ C_2 > C_3.$$

Answer: (A)

Why others are wrong: (B) claims $p_{02} = p_{03}$ (no loss in stator), incorrect when losses are considered.
(C) claims $T_{03} > T_{02}$ (stator adding stagnation temperature), impossible without work/heat addition.
(D) claims $p_{03} > p_{02}$ and equal speeds, both inconsistent with a lossy stator/diffuser.

Q43. Low-Re incompressible steady 2D airfoil characteristics depend on freestream speed, density, viscosity, airfoil chord, and angle of attack. If the objective is to achieve this with the minimum number of test runs $N_{\min}$ while taking 10 equally-spaced test values of each independent parameter, then $N_{\min}$ is ________.

(A) 10

(B) 100

(C) 10,000

(D) 1,00,000

For incompressible steady 2D aerodynamics, the key independent *non-dimensional* parameters are:

$$Re = \frac{\rho V c}{\mu}, \qquad \alpha.$$

So only **two** independent parameters need to be varied. With 10 test values each:

$$N_{\min} = 10 \times 10 = 100.$$

Answer: (B)

Why others are wrong: (A) would mean only one parameter varied; (C) and (D) treat all five dimensional quantities as independent, which is not required once nondimensionalized.

Q44. Which process(es) is/are involved in the compression of air in an **ideal ramjet** engine?

(A) oblique shock

(B) mechanical compression

(C) normal shock

(D) subsonic diffusion

In an ideal ramjet, compression occurs by **ram effect** (no compressor/turbine). Typically:

- Supersonic inlet: **oblique shocks** followed (often) by a **normal shock** to bring flow to subsonic.
- Then a **subsonic diffuser** further increases static pressure.

So the involved processes are (A), (C), and (D).

Answer: (A),(C),(D)

Why the other option is wrong: (B) Mechanical compression requires turbomachinery (compressor), which a ramjet does not have.

Q45. The deformation of an open-section bar subjected to pure torsion can be solved by choosing an appropriate Prandtl stress function. Which statements are true about the Prandtl stress function?

(A) It satisfies the equilibrium equation

(B) It is zero on the lateral surfaces of the bar

(C) It satisfies the compatibility equation

(D) It does not satisfy the equilibrium equation

For Saint-Venant torsion, the Prandtl stress function $\phi(x, y)$ is introduced such that

$$\tau_{xz} = \frac{\partial \phi}{\partial y}, \qquad \tau_{yz} = -\frac{\partial \phi}{\partial x},$$

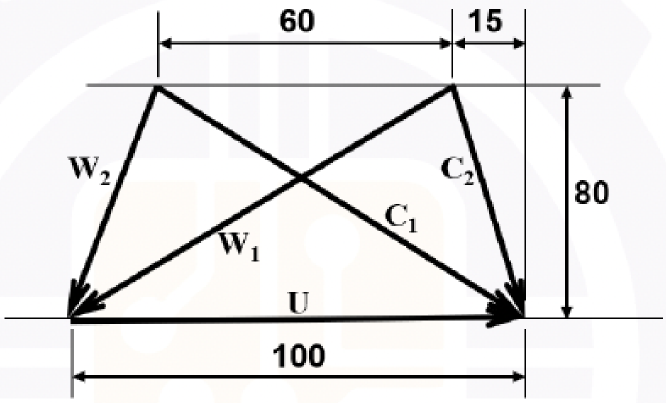
which **automatically satisfies equilibrium** (no body force):

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} = 0.$$

Also, on a free lateral surface, traction must vanish, which leads to $\phi =$ constant on the boundary; we take that constant as zero: $\phi = 0$ on the boundary.
The governing Poisson equation for ϕ comes from **compatibility + constitutive relations** in torsion.
Hence (A), (B), (C) are true.

Answer: (A),(B),(C)

Why (D) is wrong: By construction, ϕ makes equilibrium identically satisfied.

Q.46	The figure below shows the velocity triangles at the inlet and outlet of the rotor of a low hub-to-tip ratio axial compressor at the mid-span of the blade. The absolute and relative velocities of the flow are denoted by **C** and **W**, respectively. The subscripts 1 and 2 refer to the locations before and after the rotor, respectively. The blade velocity is denoted by **U**. Select the CORRECT statement(s) for this device.  All values are in m/s.
(A)	The axial velocity is constant across the rotor
(B)	The flow coefficient is 0.6
(C)	The blade loading coefficient is 0.6
(D)	If the acoustic velocity at the rotor inlet is 350 m/s, then the inlet relative Mach number is 0.333

From the given triangles (mid-span): $U = 100$ m/s and axial component $C_{a1} = C_{a2} = 80$ m/s.
(A) True: $C_{a1} = C_{a2} = 80$ m/s.
(B) Flow coefficient $\phi = \dfrac{C_a}{U} = \dfrac{80}{100} = 0.8 \neq 0.6 \Rightarrow$ false.
Tangential components read from the triangles: $C_{\theta 1} = 15$ m/s and $C_{\theta 2} = 75$ m/s. Blade loading coefficient:

$$\psi = \frac{\Delta h_0}{U^2} = \frac{U(C_{\theta 2} - C_{\theta 1})}{U^2} = \frac{C_{\theta 2} - C_{\theta 1}}{U} = \frac{75 - 15}{100} = 0.6.$$

(C) True.
Inlet relative speed:

$$W_1 = \sqrt{C_{a1}^2 + (U - C_{\theta 1})^2} = \sqrt{80^2 + 85^2} = 116.726 \text{ m/s}.$$

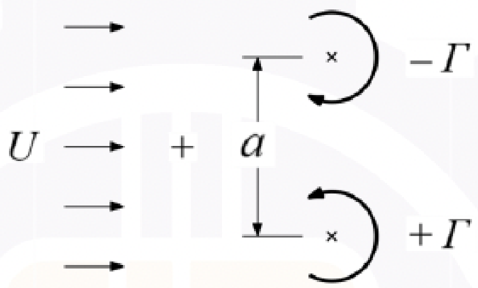
Relative Mach number:

$$M_{rel,1} = \frac{W_1}{a} = \frac{116.726}{350} = 0.3335 \approx 0.333.$$

(D) True.

Answer: (A),(C),(D)

Why (B) is wrong: It uses $\phi = 0.6$, but the triangle gives $\phi = 0.8$.

Q.47	Consider the flow over an oval modeled using the elementary potential flows as shown below. U represents uniform flow velocity and Γ represents circulation around an irrotational line vortex.  Which of the following statements is/are TRUE for this model?
(A)	Increasing U enlarges the oval
(B)	Increasing Γ enlarges the oval
(C)	Interchanging the sense of the two vortices does not alter the oval
(D)	Moving the vortices too far apart causes the oval to break up

A closed "oval" streamline exists due to a balance between uniform-flow streamfunction and vortex-induced streamfunction.
Effect of increasing Γ: stronger vortex-induced circulation strengthens the closed-streamline region, so the oval *enlarges*. Thus (B) is true.
Effect of increasing U: stronger uniform flow tends to *open up / shrink* the closed-streamline region rather than enlarge it. So (A) is false.
Interchanging sense of vortices: swapping the signs changes the induced velocity field relative to the uniform flow and generally alters the streamline pattern (it is not invariant in this configuration). So (C) is false.
Increasing separation: if vortices are moved too far apart, the interaction weakens and the single connected closed streamline can split/break up. So (D) is true.

Answer: (B),(D)

Why others are wrong: (A) stronger U suppresses closure; (C) changing sense changes the streamline topology relative to U.

Q48. What is/are the use(s) of the **single horseshoe vortex** model of finite wing aerodynamic theory?

(A) It can approximate the wing pitching moment coefficient

(B) It can approximate the wing induced drag coefficient

(C) It can approximate the effect of the wing on the induced drag coefficient of a typical horizontal tail

(D) It can approximate the aerodynamic benefit/penalty of formation flight compared to isolated flight

A horseshoe vortex model captures the **trailing vortex system** $\Rightarrow$ it is primarily useful for **downwash / induced effects**.
(B) True: induced drag comes from downwash (tilted lift), which horseshoe vortex can approximate.
(C) True: it can predict induced velocities in the far field (downwash at tail), hence estimate effect on tail induced drag.
(D) True: formation flight benefits/penalties depend on mutual induced velocities between aircraft, which can be approximated using horseshoe vortices.
(A) False: wing pitching moment coefficient needs chordwise pressure distribution / aerodynamic center effects; a single horseshoe vortex is too crude (primarily a lifting-line/spanwise model).

Answer: (B),(C),(D)

Q49. Consider the differential equation with initial conditions:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0, \qquad y(0) = 0, \qquad y'(0) = 1.$$

If $y(x)$ is the solution, the value of the slope dy/dx at $x = \ln(2)$ is __________ (rounded off to three decimal places).

Characteristic equation:

$$r^2 + 2r + 1 = (r+1)^2 = 0 \;\Rightarrow\; r = -1 \text{ (repeated)}.$$

So

$$y(x) = (C_1 + C_2 x)e^{-x}.$$

Apply $y(0) = 0 \Rightarrow C_1 = 0$. Then $y(x) = C_2 x e^{-x}$ and

$$y'(x) = C_2(1-x)e^{-x}.$$

Apply $y'(0) = 1 \Rightarrow C_2 = 1$. Thus

$$y'(\ln 2) = (1 - \ln 2)e^{-\ln 2} = \frac{1 - \ln 2}{2}.$$

Numerically, $\ln 2 = 0.693147\ldots$:

$$y'(\ln 2) = \frac{1 - 0.693147}{2} = 0.153426\ldots \approx 0.153.$$

Answer: 0.153

Q50. An object of mass 1 kg is launched with initial speed v_0 into a large tank filled with a viscous liquid. The resistive force is $D = \alpha v$, where v is instantaneous speed and $\alpha = 1$ kg/s. Gravity is ignored. The time taken by the object to slow down to speed $v_0/2$ is ________ s (rounded off to 2 decimal places).

Equation of motion (drag opposes motion):

$$m\frac{dv}{dt} = -\alpha v.$$

With $m = 1$, $\alpha = 1$:

$$\frac{dv}{v} = -dt \;\Rightarrow\; v = v_0 e^{-t}.$$

Set $v = v_0/2$:

$$\frac{1}{2} = e^{-t} \;\Rightarrow\; t = \ln 2 = 0.693147\ldots \approx 0.69 \text{ s}.$$

Answer: 0.69

Aerospace Engineering (AE) : Q51–Q60

Q51. The minimum value of the function $f(x) = |x| + |2x + 3|$ for real x is ________ (rounded off to 1 decimal place).

Rewrite
$$f(x) = |x| + |2x + 3| = |x| + 2|x + 1.5|.$$
Breakpoints are at $x = -1.5$ and $x = 0$.
Case 1: $x \geq 0$ $\quad f = x + 2(x + 1.5) = 3x + 3 \Rightarrow$ minimum at $x = 0$ gives $f_{\min} = 3$.
Case 2: $-1.5 \leq x \leq 0$ $\quad f = -x + 2(x + 1.5) = x + 3 \Rightarrow$ minimum at $x = -1.5$ gives $f_{\min} = 1.5$.
Case 3: $x \leq -1.5$ $\quad f = -x + 2(-(x + 1.5)) = -3x - 3$, which decreases as x increases, so minimum at boundary $x = -1.5$ gives $f_{\min} = 1.5$.
Hence the global minimum is 1.5.

Answer: 1.5

Q52. Isobutane (C_4H_{10}) is burnt completely in pure oxygen as per
$$C_4H_{10} + 6.5\,O_2 \rightarrow 4\,CO_2 + 5\,H_2O(v).$$
Given ΔH_f° (kcal/mol): isobutane $= -31.489$, $CO_2 = -94.052$, $H_2O(v) = -60.150$, the heat of reaction is ________ kcal (rounded off to 2 decimal places).

$$\Delta H^\circ_{rxn} = \sum \nu \Delta H_f^\circ(\text{products}) - \sum \nu \Delta H_f^\circ(\text{reactants}), \qquad \Delta H_f^\circ(O_2) = 0.$$
Products:
$$4(-94.052) + 5(-60.150) = -376.208 - 300.750 = -676.958 \text{ kcal}.$$
Reactants:
$$(-31.489) + 6.5(0) = -31.489 \text{ kcal}.$$
Therefore
$$\Delta H^\circ_{rxn} = -676.958 - (-31.489) = -645.469 \text{ kcal} \approx -645.47 \text{ kcal}.$$

Answer: -645.47

Audit note: Negative sign indicates exothermic reaction (releases 645.47 kcal per mol of isobutane).

Q53. A furnace of 250 MW rating is used to melt and raise the temperature of aluminium from 25°C to 900°C. Given: $c_{p,s} = 0.9$ kJ/kg-K, latent heat = 390 kJ/kg, $c_{p,l} = 1.108$ kJ/kg-K. Efficiency = 70%. Melting point = 660°C. Amount processed per hour is ________ kg (rounded off to 1 decimal place).

Useful power $= 0.70 \times 250 = 175$ MW $= 175$ MJ/s.
Energy per kg:
$$q = c_{p,s}(660 - 25) + L_f + c_{p,l}(900 - 660).$$

$$q = 0.9(635) + 390 + 1.108(240) = 571.5 + 390 + 265.92 = 1227.42 \text{ kJ/kg} = 1.22742 \text{ MJ/kg}.$$

Energy available in 1 hour:

$$E_{1h} = 175 \times 3600 = 630000 \text{ MJ}.$$

Mass per hour:

$$m = \frac{E_{1h}}{q} = \frac{630000}{1.22742} = 513271.7407 \text{ kg}.$$

Answer: 513271.7

Q54. Pitching moment coefficient about a reference point at $x_{ref} = 0.3c$ varies with C_l as:

C_l	0.2	0.4	0.6	0.8
$C_{m,ref}$	−0.02	0	0.02	0.04

The aerodynamic center location from the leading edge as a fraction of chord is ________ (rounded off to 1 decimal place).

For moments about a point x_{ref}:

$$C_{m,ref} = C_{m,ac} + (x_{ref} - x_{ac})\, C_l \quad \Rightarrow \quad \frac{dC_{m,ref}}{dC_l} = x_{ref} - x_{ac}.$$

Slope from table:

$$\frac{dC_{m,ref}}{dC_l} = \frac{0.04 - (-0.02)}{0.8 - 0.2} = \frac{0.06}{0.6} = 0.1.$$

Thus

$$x_{ac} = x_{ref} - 0.1 = 0.3 - 0.1 = 0.2.$$

Answer: 0.2

Q55. Satellite in elliptical orbit: perigee altitude 300 km, apogee altitude 3000 km. Earth radius $R_e = 6378$ km. Eccentricity is ________ (rounded off to 3 decimals).

$$r_p = R_e + 300 = 6678 \text{ km}, \qquad r_a = R_e + 3000 = 9378 \text{ km}.$$

Eccentricity:

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{9378 - 6678}{9378 + 6678} = \frac{2700}{16056} = 0.1682.$$

Answer: 0.168

Q56. Finite wing: $AR = 10$, span effectiveness factor $e = 0.95$. Airfoil lift slope $a_0 = 0.106$ per degree and zero-lift angle $\alpha_{L=0} = -1.5°$. Lift coefficient at $\alpha = 3.5°$ is ________ (rounded off to 2 decimals).

Convert a_0 to per radian:

$$a_0 = 0.106\ \frac{1}{\circ} \times \frac{180}{\pi} = 6.073\ \frac{1}{\text{rad}}.$$

Finite-wing lift slope:

$$a = \frac{a_0}{1 + \dfrac{a_0}{\pi e AR}} = \frac{6.073}{1 + \dfrac{6.073}{\pi(0.95)(10)}} = 5.046\ \frac{1}{\text{rad}}.$$

Effective angle:

$$\alpha - \alpha_{L=0} = 3.5 - (-1.5) = 5^\circ = 0.087266 \text{ rad}.$$

Hence

$$C_L = a(\alpha - \alpha_{L=0}) = 5.046(0.087266) = 0.4404 \approx 0.44.$$

Answer: 0.44

Q57. Maximum take-off weights W_A and W_B correspond to: Condition A: $p = 1$ bar, $T = 50°\text{C}$; Condition B: $p = 0.66$ bar, $T = -30°\text{C}$. If all other take-off parameters are same, ratio W_B/W_A is ________ (rounded off to 3 decimals).

With same take-off speed and geometry/coefficients, lift-off condition gives $W \propto \rho$. Using ideal gas $\rho = \dfrac{p}{RT}$:

$$\frac{W_B}{W_A} = \frac{\rho_B}{\rho_A} = \frac{p_B/T_B}{p_A/T_A} = \frac{p_B T_A}{p_A T_B}.$$

Convert to Kelvin: $T_A = 50 + 273 = 323$ K, $T_B = -30 + 273 = 243$ K.

$$\frac{W_B}{W_A} = \frac{0.66 \times 323}{1 \times 243} = 0.8770.$$

Answer: 0.877

Q58. Thin-walled circular tube: ultimate strength (tension/compression) = 200 MPa. Mean radius $r = 0.2$ m, thickness $t = 0.004$ m. Based on maximum stress criteria, max torque is ________ kN-m (nearest integer).

For a thin-walled closed circular tube:

$$q = \frac{T}{2A_m}, \qquad A_m = \pi r^2, \qquad \tau = \frac{q}{t} = \frac{T}{2\pi r^2 t}.$$

In pure shear, principal stresses are $\sigma_{1,2} = \pm\tau$. Maximum stress criterion $\Rightarrow |\sigma_{max}| = \tau \leq 200$ MPa. Thus

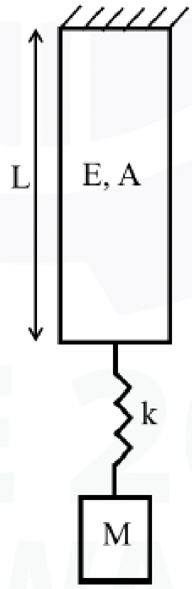
$$T_{max} = 2\pi r^2 t\, \tau_{allow}.$$

$$T_{max} = 2\pi(0.2)^2(0.004)(200 \times 10^6) = 2.01062 \times 10^5 \text{ N m} = 201.06 \text{ kN m}.$$

Nearest integer:

Answer: 201

Q.59	A system comprising a bar, spring and mass is shown in the figure below. The bar, having negligible mass, is made of a material having Young's modulus E = 200 GPa, cross-sectional area A = 100 mm^2, and length L = 100 mm. The spring stiffness k = 200 kN/mm and the mass M = 100 kg. The natural frequency of free vibration of the system is ________ rad/s (*rounded off to the nearest integer*).



Axial stiffness of bar:

$$k_{bar} = \frac{AE}{L}.$$

Using mm-units: $E = 200$ GPa $= 200000$ N/mm^2,

$$k_{bar} = \frac{(100)(200000)}{100} = 200000 \text{ N/mm} = 200 \text{ kN/mm}.$$

Since bar and spring are in **series**:

$$\frac{1}{k_{eq}} = \frac{1}{k_{bar}} + \frac{1}{k} \Rightarrow k_{eq} = \frac{k_{bar}k}{k_{bar}+k} = \frac{200\times 200}{200+200} = 100 \text{ kN/mm}.$$

Convert: 1 kN/mm $= 10^6$ N/m $\Rightarrow k_{eq} = 100 \times 10^6$ N/m.

$$\omega_n = \sqrt{\frac{k_{eq}}{M}} = \sqrt{\frac{100\times 10^6}{100}} = \sqrt{10^6} = 1000 \text{ rad/s}.$$

Answer: 1000

Q.60	A stepped cantilever beam, made of a material having Young's modulus E = 200 GPa, is shown in the figure below. The length and the moment of inertia of the beam from point A to B are L_1 = 100 mm and I_1 = 100 mm^4, respectively. The length and the moment of inertia of the beam from point B to C are L_2 = 100 mm and I_2 = 700 mm^4, respectively. A shear force P = 30 N is applied at point A of the beam. The magnitude of the deflection of the beam at point A is ________ mm (*rounded off to 1 decimal place*).

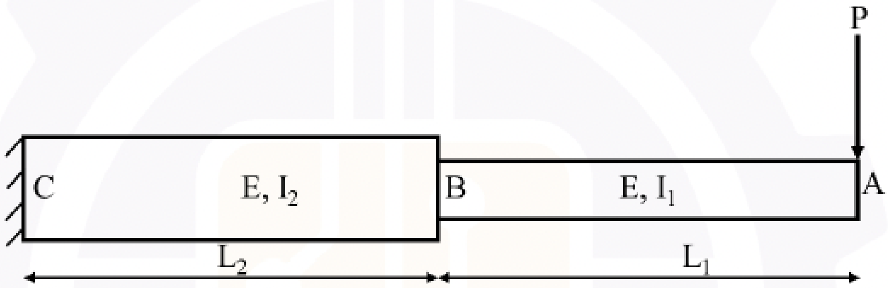


Figure 2: Enter Caption

Let z be measured from free end A towards the fixed end. Bending moment due to end load: $M(z) = Pz$ over the entire beam.
Using Castigliano / unit-load:

$$\delta_A = \int_0^{L_1+L_2} \frac{M(z)\,\partial M/\partial P}{EI(z)}\,dz = \frac{P}{E}\left[\int_0^{L_1} \frac{z^2}{I_1}\,dz + \int_{L_1}^{L_1+L_2} \frac{z^2}{I_2}\,dz\right].$$

Evaluate:

$$\delta_A = \frac{P}{E}\left[\frac{L_1^3}{3I_1} + \frac{(L_1+L_2)^3 - L_1^3}{3I_2}\right].$$

Use $E = 200$ GPa $= 200000$ N/mm^2 (consistent with I in mm^4 and z in mm):

$$\delta_A = \frac{30}{200000}\left[\frac{100^3}{3(100)} + \frac{200^3 - 100^3}{3(700)}\right] = 1.0 \text{ mm}.$$

Answer: 1.0

Aerospace Engineering (AE) : Q61–Q65

Q61. A centrifugal compressor has a constant-width radial diffuser. The diameters at the diffuser inlet and outlet are 0.2 m and 0.3 m, respectively. The flow at the diffuser inlet and outlet are assumed to be steady and uniform at the diffuser inlet and outlet and the absolute velocities at the diffuser inlet and outlet are $(60\,\hat{\mathbf{e}}_r + 75\,\hat{\mathbf{e}}_\theta)$ m/s and $(u\,\hat{\mathbf{e}}_r + 50\,\hat{\mathbf{e}}_\theta)$ m/s, respectively. If the flow at the diffuser is treated as steady and incompressible, the value of u is ________ m/s (rounded off to the nearest integer).

For steady incompressible flow in a **constant-width** radial diffuser, the flow area is

$$A = 2\pi r\, b \quad (b = \text{constant}),$$

so continuity gives

$$\rho A V_r = \text{const} \;\Rightarrow\; r V_r = \text{const}.$$

Here, $r_1 = \dfrac{0.2}{2} = 0.1$ m, $r_2 = \dfrac{0.3}{2} = 0.15$ m, and $V_{r1} = 60$ m/s, $V_{r2} = u$. Hence

$$r_1 V_{r1} = r_2 V_{r2} \;\Rightarrow\; u = V_{r2} = \frac{r_1}{r_2} V_{r1} = \frac{0.1}{0.15}(60) = 40.$$

Answer: 40

Q62. A gas mixture enters a combustor at 800 kPa and a density of 5 kg/m^3 and then enters a turbine stage. The temperature of the gas at the nozzle exit and the stage exit are 790 K and 750 K, respectively. Assume the specific heats are constant for the gas mixture in the range of temperatures considered. The specific heat at constant pressure is 0.72 kJ/kg-K and the ratio of specific heats is 1.33. The value of the degree of reaction for the turbine stage is ________ (rounded off to 2 decimal places).

First find the gas constant:

$$R = c_p \frac{\gamma - 1}{\gamma} = (0.72 \times 10^3)\frac{1.33 - 1}{1.33} = 178.65 \text{ J/kg-K}.$$

Using ideal gas relation at turbine-stage inlet (taken as stator inlet):

$$T_1 = \frac{p}{\rho R} = \frac{800000}{5(178.65)} = 895.62 \text{ K}.$$

Degree of reaction (definition):

$$R_{\text{rxn}} = \frac{\text{static enthalpy drop in rotor}}{\text{static enthalpy drop in stage}} = \frac{h_2 - h_3}{h_1 - h_3} = \frac{c_p(T_2 - T_3)}{c_p(T_1 - T_3)} = \frac{T_2 - T_3}{T_1 - T_3}.$$

With $T_2 = 790$ K (nozzle exit) and $T_3 = 750$ K (stage exit),

$$R_{\text{rxn}} = \frac{790 - 750}{895.62 - 750} = \frac{40}{145.62} = 0.2747 \approx 0.27.$$

Answer: 0.27

Audit note: This uses the standard reaction definition in terms of *static* temperature drops, and assumes inlet kinetic energy is not required to evaluate T_1 (since T_1 is obtained from p, ρ).

Q63. Thin airfoil theory predicts the zero-lift angle of attack $\alpha_{L=0}$ for NACA 2412 airfoil as $-2.1°$. The corresponding prediction of $\alpha_{L=0}$ for NACA 5410 airfoil is ________ degrees (rounded off to 1 decimal place).

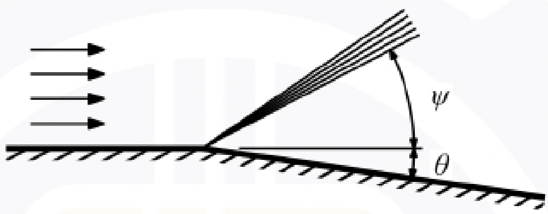
For NACA 4-digit airfoils, thin airfoil theory gives $\alpha_{L=0}$ that is **linear in the maximum camber** m for a fixed camber location p.
NACA 2412: $m = 0.02$, $p = 0.4$ and $\alpha_{L=0} = -2.1°$.
NACA 5410: $m = 0.05$, $p = 0.4$ (same p), so

$$\alpha_{L=0}(5410) = \alpha_{L=0}(2412)\left(\frac{0.05}{0.02}\right) = (-2.1) \times 2.5 = -5.25° \approx -5.3°.$$

Answer: -5.3

Q.64 Consider a centered Prandtl-Meyer expansion fan at a $\theta = 4°$ corner in a Mach 1.78 air flow, as shown in the figure below. The angle ψ (see figure) made by the ending wave of the fan with respect to the incoming stream is ________ degrees (*rounded off to 1 decimal place*).

An excerpt from the table of Prandtl-Meyer function for air is provided below.



M	*ν* [deg]	*M*	*ν* [deg]	*M*	*ν* [deg]
1.72	18.40	1.82	21.30	1.92	24.15
1.74	18.98	1.84	21.88	1.94	24.71
1.76	19.56	1.86	22.45	1.96	25.27
1.78	20.15	1.88	23.02	1.98	25.83
1.80	20.73	1.90	23.59	2.00	26.38

For a centered expansion,

$$\nu(M_2) = \nu(M_1) + \theta.$$

From the provided table, for $M_1 = 1.78$,

$$\nu(M_1) = 20.15° \;\Rightarrow\; \nu(M_2) = 20.15° + 4° = 24.15°.$$

From the same table, $\nu = 24.15^\circ$ corresponds to $M_2 = 1.92$.
The downstream Mach angle is

$$\mu_2 = \sin^{-1}\left(\frac{1}{M_2}\right) = \sin^{-1}\left(\frac{1}{1.92}\right) = 31.39^\circ.$$

The ending wave makes angle μ_2 with the *downstream* flow direction, which is turned by θ from the incoming flow. Hence, with respect to the incoming stream,

$$\psi = \theta + \mu_2 = 4^\circ + 31.39^\circ = 35.39^\circ \approx 35.4^\circ.$$

Answer: 35.4

Q.65 A Mach 1.5 air flow enters a round duct of length 20 cm and diameter 3 cm. If the flow exits with Mach number 1.1, the average Fanning friction factor f of the duct is ________ $\times 10^{-3}$ (*rounded off to 1 decimal place*).

An excerpt from the Fanno flow table for air is given below.

M	1.1	1.2	1.3	1.4	1.5	1.6
$\frac{4fL^*}{D} \times 10^4$	99.35	336.4	648.3	997.4	1361	1724

For Fanno flow, using the tabulated parameter

$$\left(\frac{4fL^*}{D}\right)(M),$$

the actual duct length satisfies (supersonic → lower Mach with friction):

$$\frac{4fL}{D} = \left(\frac{4fL^*}{D}\right)_{M=1.5} - \left(\frac{4fL^*}{D}\right)_{M=1.1}.$$

From the table (row labeled $(4fL^*/D) \times 10^4$):

$$\left(\frac{4fL^*}{D}\right)_{1.5} = 1361 \times 10^{-4}, \qquad \left(\frac{4fL^*}{D}\right)_{1.1} = 99.35 \times 10^{-4}.$$

So

$$\frac{4fL}{D} = (1361 - 99.35) \times 10^{-4} = 1261.65 \times 10^{-4} = 0.126165.$$

With $L = 0.20$ m and $D = 0.03$ m:

$$f = \frac{0.126165\,D}{4L} = \frac{0.126165(0.03)}{4(0.20)} = 0.004731 = 4.731 \times 10^{-3}.$$

Rounded to 1 decimal in the requested form:

Answer: 4.7